Reassessing Husserl's Account of the Time-continuum after the Debate on Presentism and Eternalism

Claudio Tarditi
Università di Torino

Claudio Tarditi (1978), PhD, is post-doctoral fellow in theoretical philosophy (University of Turin) and Assistant Professor in philosophical anthropology (Pontifical Salesian University). His main research field is phenomenology, especially the issues of time and intersubjectivity in Husserl's thought and their reception among French phenomenologists. More recently, his works are mainly focused on the possibility of a phenomenological foundation of physics.

The recent philosophical debate about the nature of time is peculiarly focused on the divide between presentism and eternalism. As a matter of fact, after Einstein's General Relativity theory most theoretical physicists opted for eternalism, also known as "block-universe theory". This view finds support in Minkowski's famous paper *Space and Time* (1908). Even if theoretical physicists commonly accept nowadays the concept of spacetime as a mathematical tool, the situation is much more complex for what concerns the consensus about its very nature. For instance, for Rovelli's Quantum Gravity (QG) theory our perception of space and time as *continua* reveals itself as an illusion, that is, a blurry sight of elementary processes. My aim in this paper is to demonstrate that *a)* the opposition between eternalism/spacetime theory and QG theory is rooted in their underestimation of subjective experience; *b)* such a divide could be fruitfully overcome by transcendental phenomenology, based on the idea that the very experience of time is intuitively given as a continuum; *c)* the formalization of spacetime is possible only under this basic subjective experience.

ETERNALISM SPACETIME QUANTUM GRAVITY
CONTINUUM PHENOMENOLOGY

I. Introduction

The recent philosophical debate about the nature of time is peculiarly focused on the divide between presentism and eternalism. Although there exist many variations of these theories, it could be basically argued that, according to presentism only the present is real, whereas for eternalism there are such things as merely past and future entities (Sider 1999, 325-326). As a matter of fact, after Einstein's General Relativity theory most theoretical physicists opted for eternalism, also known as "block-universe theory". This view finds support in Minkowski's famous paper *Space and Time* (1908), where he provided empirical evidence of the reality of spacetime, consistently with Einstein's theory of special and general relativity. As is well known, Minkowski opens his paper as follows: «The views on space and time which I wish to lay before you have sprung from the soil of experimental physics. Therein lies their strength. Their tendency is radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality» (Petkov 2010, xv). Against Einstein, who initially disagreed with the concept of spacetime (but accepted it in 1915), Minkowski claimed that the idea of manifolds is essential for general relativity. Indeed, «we would then have in the world no longer *the* space, but an infinite number of spaces, analogously as there are in three-dimensional space an infinite number of planes. Three-dimensional geometry becomes a chapter in four-dimensional physics» (Petkov 2010, xxi). This means that, according to Minkowski, inertial observers in relative motion may occupy different spaces, as entailed by special relativity, only in a *real* four-dimensional world. Conversely, in his eyes, special relativity does not work in a three-dimensional world, simply because this latter would imply *one* absolute space for all inertial observers. As a result, Minkowski's idea of the four-dimensional spacetime not only excludes any variety of time-flow, but directly leads to a radical eternalism.

Nevertheless, even if theoretical physicists commonly accept nowadays the concept of spacetime as a mathematical tool, the situation is much more complex for what concerns the consensus about its very nature. [1] Indeed, as emphasized by Petkov, the present situation is characterized by a «proliferation of views [...] which reject the reality of spacetime either explicitly or implicitly by defending concepts (e.g. becoming and flow of time) which are incompatible with the spacetime view of the world» (2019, 1). Among them, it is worth considering Rovelli's claim that both presentism and eternalism are "naïve options" (2019, 1325-1335) to be overcome in favor of a third way, namely a "local" account of time. In synthesis, Rovelli maintains that, following general relativity and QG (quantistic gravitation), time depends on the gravitational field: in other words, once assumed the quantum framework of the gravitational field, the quantum events are no longer ordered by a unique time. Rather than entailing that there is no change or becoming in the universe, QG demonstrates that the elementary processes cannot be ordered in a common succession of instants. Accordingly, time (as a succession of instants) does not exist; rather, time locally emerges from the relations among quantum events. There is neither a space that contains the world nor a time along which the events happen. Instead, there are elementary processes in which *quanta* of space and matter continuously interact. As a result, for Rovelli our perception of space and time as *continua* reveals itself as an illusion, that is, a blurry sight of elementary processes (2019, 1326).

1 Actually, this debate dates back to 1908, when Sommerfeld remarks in his notes on Minkowski's article: «What will be the epistemological attitude towards Minkowski's conception of the time-space problem is another question, but, as it seems to me, a question which does not essentially touch his physics».

Following Petkov's criticism, Rovelli's argument comes at a high price. First, rejecting both presentism and eternalism keeps unaddressed the question of the dimensionality of the world (at a macroscopic scale). Indeed, if presentist account of the world is three-dimensional and evolving in time and eternalism (also called "block universe") accounts for a four-dimensional world with time as fourth dimension, claiming (as Rovelli does) «neither presentism nor eternalism» should entail that the world is neither three-dimensional nor four-dimensional. This means that, in Rovelli's eyes, dimensionality is not a fundamental feature of the world, as commonly accepted in physics. Secondly, Petkov assesses Rovelli's account of eternalism as a complete misrepresentation. Indeed, rather than claiming «that past and future events are "real now" as present events» (2019, 1329), the spacetime theory is based on the thesis that no event is to be privileged as "now", insofar as in spacetime the notion of «real now» is meaningless. In other words, Minkowski introduced a static view of time for which spacetime is an unchanging, once-and-for-all picture of the world encompassing past, present, and future.

Accordingly, from the eternalist perspective, there is no privilege of the "now" and, thus, it is totally illegitimate to talk about past and future events simply because they are equally real. It is worth emphasizing that, what is really at stake in Rovelli's attempt to overcome eternalism and Petkov's defense of Minkowski's spacetime is a precise description of what time really is, namely an ontology of time. In other terms, both spacetime and QG theory elude any inspection of how the experience of time (as becoming and flow) interacts with the physical (real) features of time. If, on the one hand, Minkowski admits that the reality of spacetime is a counterintuitive picture of the world, albeit supported by a number of experimental evidences, on the other hand, Rovelli argues that, all things considered, time's continuity is a persisting illusion. In other terms, they both underestimate the fact that *time is always experienced as a continuum*. Indeed, as stressed above, the conflict between spacetime theory and our common experience of time extensively characterizes the history of both contemporary theoretical physics and philosophy of time.

With this regard, such a conflict is probably at the root of the very diffused interpretation of spacetime uniquely as a mathematical tool. However, some eminent physicists felt the need to reconcile spacetime with the subjective experience of time. Among others, H. Weyl emphasized that «the objective world merely exists, it does not happen; as a whole it has no history. Only before the eye of the consciousness climbing up in the world line of my body, a section of this world "comes to life" and moves past it as a spatial image engaged in temporal transformation» (Weyl 2009, 135). In this short passage, there clearly emerges how, according to Weyl, the reality of spacetime and the subjective experience of the time's flow are to be understood on two different levels. Indeed, if from an ontological point of view the world does not happen, in the subject's eyes reality ceaselessly transforms itself throughout an infinite process. Thus, Weyl seems to suggest that the problem of time cannot be merely addressed from the ontological standpoint, but also requires an in-depth inspection of the subject's experience of temporal becoming. In other words, both theoretical physicists and philosophers of time should deal not only with the description of the ontological features of spacetime, but also with the transcendental inspection of the experience of the continuum as an absolute given for any subject.

Under these premises, my aim in this paper is to demonstrate that *a)* the opposition between eternalism/spacetime theory and QG theory is rooted in their underestimation of subjective experience; *b)* such a divide could be fruitfully overcome by transcendental phenomenology, based on the idea that the very experience

of time is intuitively given as a continuum; *c)* the formalization of spacetime is possible only under this basic subjective experience.

II. Ontology and Epistemology of Time

As is well known, Weyl's early account of time-continuum (developed in his major works *The Continuum*, 1987, and *Space Time Matter* 1922, both published in 1918) is deeply influenced both by neokantianism [2] and Husserl's transcendental phenomenology. [3] Such an influence is to be essentially recognized in Weyl's claim that coordinate system is «the unavoidable residuum of the ego's annihilation» (1987, § 5.3.4), a clear reprise of Husserl's account of transcendental ego as the residuum of phenomenological reduction in *Ideen I* (1976, § 49). Nevertheless, although Weyl always refers to *Ideen I*, his account of the continuum is deeply indebted to Husserl's lectures on time-consciousness and the *Dingvorlesungen* (1997) on the constitution of space, some of which he probably attended when Husserl substituted for Hilbert (who was Weyl's teacher of mathematics at that time) during the years 1905-'08. In Weyl's view, our intuition about the continuum originates from common or stable elements, namely invariants emerging from a plurality of acts of experience: for instance, the perception of time, of movement, of a line extended, and so forth. For what concerns time, Weyl considers Husserl's (and Bergson's) phenomenal time as a conscious experience coexisting with memory of the instant just gone. Consistently with Husserl's perspective, Weyl describes the intuition of time as a flow of ongoing transformations. This means that time is a duration without points: time consists in interconnected parts that are superimposed on each other. With this respect, Weyl's phenomenological heritage is patent:

[2] See Bernard & Lobo (2019).

[3] For more biographical and theoretical details, see Feist (2004).

> The view of a flow consisting of points and, therefore, dissolving into points turns out to be false. Precisely what eludes us is the nature of the continuity, the flowing from point to point; in other words, the secret of how the continually enduring present can continually slip away into the receding past. (1987, 91-92).

In other terms, Weyl shares with Husserl a radical opposition between, on the one hand, time and space as pre-phenomenal experiences and, on the other hand, time and space as *construed* mathematical entities. This means that, whereas the construed entities resulting from mathematical formalization are made out of ultimate elements (the points), the pre-phenomenal life-experiences –time in particular –cannot be further reduced. More closely, Weyl accepts Cantor and *Dedekind*'s axiom about the one-to-one correspondence between the real line and the pre-phenomenal space, but judges unsatisfactory the extension of such a correspondence to time. Also considering spatial movement, the situation is not significantly different. Indeed, «in movement, the continuum of points on a trajectory recovers in a continuous monotone fashion the continuum of instants» (Weyl 1987, § 8). But following Weyl's argument, this is just superposition: the temporal continuum does not have points, the instants are merely transitions, the present is only possible because of the simultaneous perception of the past and of the future. Accordingly, Weyl maintains:

> I think that everything we are demanding here is obvious nonsense: to these questions, the intuition of time provides no answer - just as a man makes no reply to questions which clearly are addressed to him by mistake and, therefore, when addressed to him, are unintelligible. So the theoretical clarification of the essence of time's continuous flow is not forthcoming. The

> category of the natural numbers can supply the foundation of a mathematical discipline. But perhaps the continuum cannot [...]. (1987, 90).

From this there follows that 1) an individual point in time is non-independent, i.e., is pure nothingness when taken by itself, and exists only as a "point of transition" (which, of course, can in no way be understood mathematically); 2) it is due to the essence of time (and not to contingent imperfections in our medium) that a fixed time-point cannot be exhibited in any way, that always only an approximate, never an exact determination is possible (Weyl, 1987, 92). Points do not belong to our intuition of the continuum, neither temporal nor spatial. [4]

As a result, «the point without dimensions is a derived conceptual construction, a necessary consequence of a line as a one-dimensional law. It is a posterior reconstruction [...] which puts together the points to reconstruct the line» (Longo 1999, 404). In the light of this inspection, it could be argued that Weyl's phenomenological inspiration leads him to account for the transcendental features of time experience as fundamental issues for any theory of time. Accordingly, although the evidence-based concept of spacetime provides with a useful mathematical tool for sketching an ontology of time, it cannot explain the subjective experience of time at all. Thus, from Weyl's perspective a complete theory of time is supposed to think *together* the ontology of time and the epistemology of time experience. In other words, there is no ontology of spacetime beyond an epistemological enquiry on time experience.

4 A precise proof of the fact that a curve is a *law*, not a set of points, is provided by Wittgenstein (1964).

Nevertheless, Weyl's phenomenological posture is not so radical as it seems at first glance. Indeed, his thought leads to a precise distinction between objective spacetime and subjective time experience, rather than an in-depth assessment of their actual interaction. As suggested by R. A. Feist (2004, 138), Weyl's declaration of phenomenological membership should not prevent from considering two points of substantial divergence. *a)* Firstly, Weyl's claim in *The Continuum* that the sequence of natural number is given in an immediate intuition of iteration is clearly in contradiction with Husserl's view that there cannot be any direct access to formal categories (included the sequence of natural numbers), uniquely graspable by categorial acts. *b)* The second point of divergence concerns Weyl's choice for predicativism. Following Weyl's argumentation in chapter 6 of *The Continuum* (probably influenced by Poincaré and Russell), the constitution of all higher level objectivities depends on the immediate intuition of natural numbers. Thus, whereas the latter exist independently, all other objectivities are constituted in conformity to logical constraints, that is, in Weyl's view, predicative constructions based upon the domain of natural numbers. By contrast, in Husserl's thought such logical restrictions are totally excluded, insofar as he was firmly convinced that mathematics is a variety of knowledge of reality and, at the same time, a formal theory of possible regions of being. In other terms, Husserl was not committed to any kind of restriction of mathematics with the purpose of fitting it into strictly intuitive (or predicative) limits. Rather, he was strongly involved in a research of formal systems, as testified by the *Mannigfaltigkeitslehre* developed through the *Logical Investigation* as a part of his formal ontology. This means that Husserl's phenomenology does not exclude any formal (non-intuitive) mathematics, as well as formal and axiomatic theories of analysis capable of providing a ground for regional ontologies.

These two arguments lead Weyl to the thesis that a phenomenological foundation of the mathematical continuum based upon the analogy between the

immediate intuition of time and the intuition of the natural numbers succession is in principle excluded. Once demonstrated that the intuitive and the mathematical continuum do not coincide, Weyl suggests that its mathematical construction must necessarily overcome the level of phenomenological description. [5] This is the basic reason why Weyl progressively shift from phenomenology to predicativism. With this regard, in a lecture delivered in Princeton in 1927, with the title *Time Relations in the Cosmos, Proper Time, Lived Time, and Metaphysical Time*, Weyl concludes:

> The immediately experienced is subjective and absolute. On the other hand, the objective world is necessarily relative and may be represented by something definite, numbers or other symbols, only after a coordinate system has been arbitrarily imposed on the world. This pair of opposites, subjective-absolute and objective-relative, seems to me one of the most fundamental epistemological insights one can gather from science. (Weyl 2009, 31)

[5] As stated by Da Silva (1997, 289): «Despite its debt to certain phenomenological ideas, the system of *The Continuum* cannot be seen as a prototype of how the whole of mathematics should be developed from the phenomenological perspective».

III. Time, Reality, and the Transcendental

With and beyond Weyl, is it possible to conceive of spacetime and time experience *together*? More precisely, once grasped the double nature of time, how to sketch a description of the bond of ontology and epistemology of time? In other words, how to take into account both the physical evidence of spacetime and the experience of the flowing time *as two absolute evidences*? My core argument is that this attempt should be made through an in-depth reassessment of Husserl's transcendental phenomenology.

Within the limit of this work, I cannot provide a complete discussion of the huge quantity of passages –from the published texts and manuscripts –where Husserl deals with the issue of the continuum. I will limit to the most insightful. Although the continuum, namely the phenomenological condition of both the flux of the lived-experiences and the flowing of the intuitive data, is a real *leitmotiv* of the phenomenological method as a whole, it plays a peculiar role in the early Husserl, notably in his lectures of 1891 on *Philosophy of Arithmetic* (2003), those of 1905–1908 *On the Phenomenology of the Consciousness of Internal Time* (1991), and those of 1907 on *Things and Space* (1997). As emphasized by Tieszen (1996, 304), «Husserl thinks that arithmetical knowledge is originally built up in founding acts from basic, everyday intuitions in a way that reflects our a priori cognitive involvement». [6] Within this framework, it is worth noting how Husserl takes into serious consideration both the intuitive and formal structure of the continuum since his first great work. For instance, let us consider the following passage from the section about the *Origin of the Concept of Manifold* in *Philosophy of Arihmetics*:

> If we consider, for example, the cohesion of the points on a line, of the moments of a span of time, of the color nuances of a continuous color spectrum, of the tonal qualities in a "tone progression", and so on, then we acquire the concept of combination-by-continuity, and, from this concept, the concept of the continuum. This latter concept is not contained as a particular, distinguishable, partial content in the image of every concretely given continuum. What we note in the concrete case is, on the one hand, the points or extended parts, and, on the

[6] See also Centrone (2010, 1) (footnote 2).

other hand, the peculiar combinations involved. These latter, then, are what is always identically present whenever we speak of continua, however different may be the absolute contents which they connect (places, times, colors, tones, etc.). Then in reflection upon this characteristic sort of combination of contents there arises the concept of *continuum,* as that of a *whole* the parts of which are united precisely in the manner of continuous combination. (2003, 20) 7

7 A very similar passage is to be found in the *Third Logical Investigation*: «Two contemporaneous sensuous *concreta* necessarily form an 'indifferentiated whole' if all the immediately constitutive 'moments' of the one pass unbrokenly over into corresponding constitutive 'moments' of the other. The case of exact likeness of any such corresponding moments shall count as a legitimate limiting case of continuity, i.e. as a continuous "passing over into self" » (Husserl, 2001b, 14).

It is worth putting this passage in connection with Husserl's discourse on mathematical entities in § 60 of the *Sixth Logical Investigation* (2001c), where he distinguishes between sensuous abstraction and pure categorial abstraction. Whereas sensuous abstraction gives sensuous concept (for instance, "house, red") and mixed concepts, categorical abstraction gives categorical concepts (for instance, "relation, set, number"), called by Husserl "formal-ontological categories". If sensuous and mixed concepts are based upon sensuous intuitions, categorical concepts depend on categorical intuitions. Concerning the categorical intuition of a set, categorical abstraction refers to the collection's form, without any consideration of all material aspects of the set's members. Accordingly, provided that logico-mathematical intuition is a categorical intuition purified by categorical abstraction, pure logic and mathematics include no sensuous concepts. Once intuitively grasped a mathematical concept, one can grasp other mathematical objects in new categorial acts of higher level. Thus, mathematics results being based upon pure categorial abstraction, which excludes all the material contained in the categorial intuition.

From this standpoint, there emerges how the concept of the continuum originates in intuition of concrete data: more precisely, the intuition of continuity is the phenomenological condition for any mathematical formalization of the continuum. This does not entail that Husserl is not committed to the problem of a rigorous formalization of the continuum. With this respect, it must be noted a strong influence of Hilbert's view, following which «one begins by assuming the existence of all elements (that is one assumes at the beginning three different systems of things: points, lines and planes) and one puts these elements into certain relations to one-another by means of certain axioms, in particular the axioms of connection, order, congruence and continuity» (Hilbert 1900, 181). Nevertheless, Husserl is fully aware of the limits of any attempt of formalization of the continuity (the same limits Weyl will emphasize concerning Cantor-*Dedekind*'s axiom):

We are able to bring *each single* group element to representation in its own right in temporal succession, even though not in one allinclusive act. But all of this is impossible in the cases to which we now turn. We speak of totalities, groups, and multiplicities also where the concept of their authentic formation, or of their symbolization through sequential exhaustion of the individuals involved, already contains a logical impossibility. We speak of *infinite groups*. The extensions of most general concepts are infinite. The group of the numbers in the symbolically expanded number series is infinite, as is the group of points in a line, and, in general, that at the limits of a continuum. The thought that some conceivable expansion of our knowledge capacity could enable us to have the actual representation –or even the mere sequential exhaustion –of such groups is unimaginable. Here even our power of idealization has a limit. (Husserl 2003, 231)

One could spot the same tension between intuition and formalization of the continuum also in Husserl's lectures of 1907 on *Thing and Space* as well as in his courses on time-consciousness of 1905–19085. In both cases, I cannot provide even a synthetical overview of the enormous critical literature on these texts: thus I will only recall some quotations in order to show how the question of the continuum is at the very core of the general problem of the temporal and spatial perception. At the beginning of § 19 of *Thing and Space*, Husserl argues:

> Here I have in mind the wonderful phenomenological forms of appearance which have the character of extensions of appearance: in them is constituted the spatial and temporal expanse that belongs to the essence of thingly objects; in them therefore lies the source of all spatial-temporal predicates. (1997, 51)

For what concerns the spatial continuity, Husserl distinguishes two main meanings: 1) The continuity that belongs to spatial extension as such and that comes to consciousness as an immanent moment when we allow unchange to pass over into change, for example in the continuous migration of a qualitative discontinuity over an expanse filled up unitarily in such and such a way. 2) The continuity of the filling determinations themselves, for example the flowing over from quality to quality, perhaps in the transition from red through purple to violet. But what is particularly relevant for us is that, in Husserl words, «continuity is extension, and qualitative continuity qualitative extension. *That essentially implies fragmentability and the ideal possibility of an abstract differentiation into phases*. [...]» (Husserl 1997, 59). This idea of the priority of intuitive continuum upon its mathematical construction is explicitly attested by the following passage:

> Although in fact every body can be resolved into an infinite manifold of plane sections and can be considered a continuum of plane sections, yet the geometry of plane figures, which encompasses all these sectional figures, is still not the geometry of the spatial body. In proceeding beyond the plane, what is at issue is precisely the laws according to which the planes and the formations lying on them are continually modified. (1997, 173)

Much more complex appears to be the issue of temporal continuum precisely because of its irreversibility:

> If time thus appears as an eternal stream which precipitates everything temporal into the abyss of the past, yet, on the other hand, time has validity as an eternal and fixed form, since every being maintains its position in time. Even a god cannot alter the temporal positions of events in the past. Here reside immense difficulties, which up to now have defied the acumen of the greatest. We will still devote efforts of our own to these difficulties. (1997, 55)

In order to make sense of these great difficulties, one has to address the question of the continuum within the framework of Husserl's lectures on time-consciousness from 1905–1908. Although the transcendental experience of time reveals some relevant differences with respect to the perception of space, it must be admitted that, in both the lectures on time-consciousness and spatial perception, there emerges the idea of a basic impossibility of reducing the intuitive continuum phases into a set of points. [8] From this perspective, in my interpretive hypothesis, Husserl's decision of keeping the notion of the "original impression", [9]

[8] Indeed, Husserl argues that «What we call original consciousness, impression, or even perception, is an act that is shaded off continuously. Every concrete

although not conceivable as a punctual source of the temporal continuum, strictly depends on his attempt to clarify the conditions of the mathematical formalization of time. With this regard, Husserl argues:

> It is inherent in the essence of every linear continuum that, starting from any point whatsoever, we can think of every other point as continuously produced from it; and every continuous production is a production by means of continuous iteration. [...] The primal impression is the absolute beginning of this production, the primal source, that from which everything else is continuously produced. *But it itself is not produced*: it does not arise as something produced but through *genesis spontanea*; it is primal generation, it does not spring from anything. It is primal creation. (1991, 106)

perception implies a whole continuum of such shadings. But reproduction, phantasy-consciousness, also requires precisely the same shadings, only reproductively modified. It belongs to the essence of both of these experiences that they must be extended in such a way that a punctual phase can never exist by itself » (1991, 49).

9 As is known, the most serious difficulty entailed by the concept of the original impression is the infinite regression from what is constituted to what is constituting.

Accordingly, the problem of the "original impression" contains and includes the ambivalence of the intuitive (pre-phenomenal) continuum and the mathematical continuum as a logical construction. As is well known, Husserl founds these different experiences of the continuum on two varieties of intentionality (but we know from a number of manuscripts that he was not really satisfied by this solution). Expressed in terms of the double continuum of "transverse" and "lengthwise" segments, the distinction between constituted transcendent object and constituting time-consciousness designates the transverse intentionality (phase-continuum), whereas the distinction between constituted time-consciousness (immanent unity of act and its content) and constituting absolute time-consciousness designates the "lengthwise" intentionality (stretch-continuum). Nevertheless, in a manuscript from 1908 or 1909, Husserl becomes definitively aware of the impossibility of conceiving of the phenomenological continuum of time as something objective:

> Is it inherently absurd to regard the flow of time as an objective movement? Certainly! On the other hand, memory is surely something that itself has its now, and the same now as a tone, for example. No. There lurks the fundamental mistake. The flow of the modes of consciousness is not a process; the consciousness of the now is not itself now. [...] Memory is an expression that always and only refers to a constituted temporal object. Retention, on the other hand, is an expression used to designate the intentional relation (a fundamentally different relation) of phase of consciousness to phase of consciousness and in this case the phases of consciousness and continuities of consciousness must not be regarded as temporal objects themselves. These are extremely important matters, perhaps the most important in the whole of phenomenology. (1991, 345–346)

Husserl approaches anew these «extremely important matters» in the *Bernauer Manuskripte*, where the problem of the originary impression develops into the issue of individuation as a temporal process. From a genetic point of view, it is precisely in this process that the ego originates in its immanence. This means that individuation concerns not only the objects, but also the ego as such. In other words, the immanent "living-present" is the most originary type of individuation, composed of a multiplicity of sensible given unified in a continuous sequence. After a decisive discussion with R. Ingarden about the problem of the relation between the unity of sensible *data* and the flow of consciousness, Husserl recalls into question the scheme apprehension/content of apprehension introduced in his *Logical Investigations*, as testified by the

manuscripts n. 6 and 9. In these texts Husserl argues that the flux of absolute consciousness constitutes the *Erlebnisse* as temporal objects within immanent temporality. This implies that the constitution of the temporal objects is inseparable from the constitution of a temporal consciousness. In other words, the flux of absolute consciousness implies an essential correlation between immanent perception and perceived object. Accordingly, the immanent temporality, namely the noetic side of intentionality, derives from the temporal constitution of the flux of consciousness as a continuous and consistent flux. From this viewpoint, both objective and immanent temporality are based on the "originary process" (*Urprozess*) of individuation. Indeed, Husserl states in the text n. 9:

> I mean that it is only by virtue of the coincidence (*Deckung*) which crosses retention and protention and continues from an originary presentation to a new one as a coincidence of this persisting sound (where the last originary presentation falls into retention), that we grasp the sound as a temporal object. If we abstractively isolate an originary presentation and its flow in the temporal flux, we obtain in each point a new nuclear given (*Kerndatum*) taken abstractively, not a temporal objective given (*zeitgegenstandliches*). In other terms, we will obtain no representation (*Darstellung*) of something objective within the nuclear given. Accordingly, the persisting perception of a sound is not to be understood merely as the objective series of the originary presentations. This series is constituted [...], as well as the series of the originary flux; nevertheless its objectivation has a different sense [...]. (2001a, 171) **10**

10 Translation is mine. See also Schnell (2002).

In the light of this passage, it is clear that in these manuscripts Husserl is no longer committed to the idea of an originary presentation nestled in a double horizon of retentional and protentional phenomena. Thus, he changes his perspective and emphasizes the role played by both the protensions of the flux of retentional modifications and the influence of retentions in the determination of protentional contents. It follows that the originary process (*Urprozess* or *Urstrom*) reveals itself not as a mechanism of constant modification of the present in the just passed; rather, the temporal flux is deeply intertwined with passive syntheses, anticipations and more or less intense degrees of fulfillment (*Erfullung*) and emptying (*Entfullung*). With this respect, Husserl maintains: «Differently from the previous texts, what is at stake here is no longer the mere [...] retentional consciousness of the originary flux, but rather a self-consciousness of the originary flux originally anchored in the fluent present» (Husserl 2001, XIII). As a consequence, in the *Bernauer Manuskripte* Husserl testifies his awareness of the danger of infinite regression and describes the present itself as fluent continuity: «A fluent consciousness structured in this way is necessarily a consciousness of itself as fluent» (Husserl 2001a, 48).

Still, the problem is not completely solved as long as the *Urprozess* is understood as an independent level of intentional consciousness, responsible for both the constitution of the temporal (immanent) objects and the acts of apprehension. What remains definitely open is the question of the flux's nature: provided that the flux is placed at a transcendental level, why does it manifest itself *a posteriori*, as a condition of possibility of the constituted time? If the originary process needs to be grasped by the ego, without whom the function of constitution would not be possible, its independence is seriously compromised. This means that, although the nuclear model seems to be more fruitful than the model of apprehension (always subject to the danger of infinite regression), Husserl does not fully succeed in dealing with the difference between act-consciousness and consciousness of originary consciousness.

Some scholars [11] conceive of the originary flux as the unique non-constituted element in Husserl's phenomenology. According to this view, the originary flux, understood as longitudinal intentionality, should be the origin of all temporal constitutions. Nevertheless, Husserl emphasizes how longitudinal intentionality is at the same time transversal intentionality, which is always in connection with the time of immanent objects. It follows that, in front of the problem of time, the notions of origin, process and constitution fall into a kind of short circuit. In order to summarize, what is at stake in Husserl's scrutiny of time-consciousness is the possibility of new originary presentations within the continuum of intentional givenness. More precisely, Husserl struggles to find an equilibrium between the continuous process of temporalization and the emergence of punctual new instants, that is, between the intuitive experience of continuity and the attempt to formalize it. Each protentional instant is never fully anticipated by the previous one: this means that the grasping of temporal flux does not consume the surprise of our consciousness in front of the presentation of each new instant. It is for this reason that Husserl describes consciousness as what emerges from the awakening of time itself (Husserl, 1966, 178).

11 See for instance Sokolowski (1964).

IV. Conclusions

As already pointed out, the relevance of the phenomenological discourse about the time continuum has been largely neglected by mathematicians, theoretical physicists, and philosophers of time. Regardless of their reciprocal differences, all of them developed a number of ontologies of time (i.e. spacetime theory, QG, presentism, eternalism, etc.) without taking seriously into account the issue of the subjective-transcendental time experience, often dismissed as an illusion. Especially after the demonstration of the independence of the continuum hypothesis from the set theory's axioms, this issue is nowadays rather controversial. Its history [12] has determined a global reconsideration of the notion of solution in set theory (and mathematics), because of its strong dependence on the issues of consistency and indeterminacy. With this regard, the continuum hypothesis involves a manifold of philosophical questions dealing with the question "what is a solution?" Has the continuum problem been resolved? If so, which solution has been found? Otherwise, which is its current status? Under Gödel's theorem of incompleteness, is it unavoidable a pluralistic view about the continuum?

12 For a precise overview of this discussion, see Linnebo (2017), especially chapters 4, 8, and 12.

Gödel himself took part in this debate with an article (1947) in which he claimed that, once assumed the correctness of the set theory axioms, there follows that concepts and theorems describe a particular reality for which Cantor's conjecture is either true or false. Thus, the axioms' indeterminacy implies that they do not contain a complete description of that reality (Gödel 1947). Gödel's perspective can be included in the *platonic* approach to mathematics, namely the view for which mathematics has to deal with a realm of objects and concepts independent of our mind. From this perspective, the continuum hypothesis has a given value of truth, independently of our ability to discover it. By contrast, Cohen (1963) maintains that the demonstration of the continuum hypothesis' independence of Zermelo-Fraenkel's axioms (plus the axiom of choice) [13] is completely satisfying: rather than requiring the understanding of any mathematical reality (as argued by Gödel), the solution of the continuum problem depends on the results we

13 There exist two classic formulations of the continuum hypothesis: each infinite subset of the continuum (i) has the cardinality of

can reach within a certain axiomatic system. His *formalistic* solution is widely diffused among mathematicians.

Analogously, as I emphasized above, most physicists consider spacetime theory a fruitful mathematical tool, rather than an exhaustive explanation of the very nature of time. Those (like Weyl) who grasped the relevance of the subjective experience of time ended up admitting the immeasurableness of the intuition and formalization of the time continuum. What I attempted to demonstrate in this text is that Husserl's phenomenology is still in a position to provide a rigorous description of how the spacetime is given to a transcendental subject. More closely, rather than being a mere illusion, the intuitive experience of time continuum is an absolute given without which any mathematical formalization, therefore any physical theory, would be impossible. Much work is yet to be done in this direction. Nevertheless, Husserl's thought has still much to offer in order to shed light into the enigmatic entrenchment of our transcendental life with any scientific theory.

the natural (countable infinite) or real number set; (ii) has the smallest transfinite cardinality after the countable infinite. These two formulations are equivalent if one assumes the nine-item list of rules called Zermelo-Fraenkel set theory plus the axiom of choice (1904), affirming that, given any collection of non-empty sets, it is possible to make a selection of exactly one object from each set, even if the collection is infinite.

References

Bernard, J. & Lobo, C. (Eds.) (2019). *Weyl and the Problem of Space*. Dordrecht: Springer.

Cantor, G. (1878). Ein Beitrag zur Mannigfaltigkeitslehre. *Journal fur die reine und angewandte Mathematik*, 84, 242–258.

Centrone, S. (2010). *Logic and Philosophy of Mathematics in the Early Husserl*. Dordrecht: Springer.

Claesges, L. (1964). *Edmund Husserls Theorie der Raumkonstitution*. The Hague: Nijhoff.

Cohen, P. J. (1963). The Independence of the Continuum Hypothesis. *Proceedings of the National Academy of Sciences USA*, I (50), 1143–1148.

Da Silva, J. J. (1997). Husserl's Phenomenology and Weyl's Predicativism. *Synthese*, 110, 277–296.

De Warren, N. (2009). *Husserl and the Promise of Time*. Cambridge: Cambridge University Press.

Feist, R. (Ed.). (2004). *Husserl and the Sciences. Selected Perspectives*. Ottawa: Ottawa University Press.

Føllesdal, D. (1999). Gödel and Husserl. In J. Petitot & F. Varela (Eds.), *Naturalizing Phenomenology* (385–400). Stanford: Stanford University Press.

Gödel, K. (1938). The Consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis. *Proceedings of the National Academy of Sciences USA*, 24, 556–557.

Id. (1944). Russell's Mathematical Logic. In S. Feferman, J. Dawson & S. Kleene (Eds.), *Journal of Symbolic Logic* (119–141). Evanston: Northwestern University Press.

Id. (1947). What is Cantor's Continuum Problem? *American Mathematical Monthly*, 54, 515–525.

Id. (1990). *Collected Works II: Publications 1938–1974*. Oxford: Oxford University Press.

Id. (1995). *Collected Works III: Unpublished Essays and Lectures*. Oxford: Oxford University Press.

Hilbert, D. (1900). Uber den Zahlbegriff. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 8, 180–183.

Hopkins, B. (2011). *The Origin of the Logic of Symbolic Mathematics. Edmund Husserl and Jacob Klein*. Bloomington: Indiana University Press.

Husserl, E. (1966). *Analysen zur Passiven Synthesis* (Hua XI). Den Haag: Nijoff.

Id. (1970). *The Crisis of European Sciences and Transcendental Phenomenology*. Evanston: Northwestern University Press.

Id. (1976). *Ideen zu einer reinen Phanomenologie und phanomenologischen Philosophie. Erstes Buch* (Hua III/1). Den Haag: Nijhoff.

Id. (1991). *On the Phenomenology of the Consciousness of Internal Time (1893–1917)*. Dordrecht: Kluwer Academic.

Id. (1997). *Thing and Space. Lectures of 1907*. Dordrecht: Springer.

Id. (2001a). *Die Bernauer Manuskripte uber das Zeitbewusstsein (1917/18)*. Dordrecht: Kluwer.

Id. (2001b). *Logical Investigations* (I-II). New York: Routledge.

Id. (2001c). *Logical Investigations* (III-VI). New York: Routledge.

Id. (2003). *Philosophy of Arithmetic*. Dordrecht: Springer.

Linnebø, O. (2017). *Philosophy of Mathematics*. Princeton: Princeton University Press.

Longo, G. (1999). The Mathematical Continuum. From Intuition to Logic. In J. Petitot & F. Varela (Eds.), *Naturalizing Phenomenology* (401–428). Stanford: Stanford University Press.
Petkov, V. (Ed.) (2010). *Minkowski Spacetime: A Hundred Years Later*. Dordrecht: Springer.
Id. (2019). *Neither Presentism nor Eternalism: At What Price?*. http://www.minkowskiinstitute.org/conferences.
Rovelli, C. (2019). "Neither Presentism nor Eternalism". *Foundations of Physics*, 49, pp. 1325-1335.
Rykman, T. (2005). *The Reign of Relativity*. Oxford: Oxford University Press.
Schlipp, P. A. (Ed.). (1963). *The Philosophy of Rudolf Carnap*. Chicago: Open Court.
Schnell, A. (2002). Das Problem der Zeit bei Husserl. Eine Untersuchung uber die husserlschen Zeitdiagramme. *Husserl Studies*, 18 (2), 89–122.
Sokolowski, R. (1964). *The Formation of Husserl's Concept of Constitution*. Den Haag: Nijhoff.
Tieszen, R. (2005). *Phenomenology, Logic and the Philosophy of Mathematics*. Cambridge: Cambridge University Press.
Van Dalen, D. (1984). Four Letters from Edmund Husserl to Hermann Weyl. *Husserl Studies*, 1, 1–12.
Weyl, H. (1987). *The Continuum: A Critical Examination of the Foundations of Analysis*. New York: Dover.
Id. (2009). *Mind and Nature*, Princeton: Princeton University Press.
Wittgenstein, L. (1964). *Philosophical Remarks*. Oxford: Basil Blackwell.
Zermelo, E. (1904). Beweis dass jede Menge wohlgeordnet werden kann. *Mathematische Annalen*, 59, 514–516.